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On the scalar scattering by a strip in a dissipative medium

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SUMMARY

An approximate theory of the scalar scattering by a strip in a dissipative medium is established. The theory is suitable when the relations $1/\mathrm{Im}(k) \ll h \ll d$ are satisfied by the width of the strip ($2h$), the smallest distance between the plane of the strip and the exciting sources (d) and the wave number (k). The expressions valid in the far-field approximation are obtained by simple manipulations. Some illustrative examples concerning the scattering of electromagnetic waves, emitted by Hertzian dipoles, by unidirectionally conducting strips are given.

1. Introduction

Consider the following classical boundary-value problem related to the strip S defined by $x=0$, $y \in (-\infty, \infty)$, $z \in (-h, h)$:

$$\Delta u + k^2 u = 0\,, \qquad (x, y, z) \notin S \tag{1}$$

$$\mathscr{L}_s\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) u = \psi(y, z), \qquad (x, y, z) \in S\,, \tag{2}$$

where k^2 is a complex constant, $\psi(y, z)$ is a known function defined on S while u is to be found. The linear differential operator $\mathscr{L}_s$ is a polynomial (with constant coefficients) with respect to the symbols appearing in the argument, such that

$$\left(\frac{\partial}{\partial x}\right)^2 u = \frac{\partial^2 u}{\partial x^2}, \quad \left(\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial y}\right) u = \frac{\partial^2 u}{\partial x \partial y} \text{ etc.}$$

In addition to Eqn. (2), one stipulates also that u satisfies the radiation condition at infinity and some edge condition at the edges of S.

Although this problem has been subject to a great deal of investigations in case of plane incident waves [see, for example, 1–5], a general approximate theory valid for arbitrary incident waves is still lacking. The aim of this paper is to establish such a theory when the relations $1/\mathrm{Im}(k) \ll h \ll d$ are fulfilled, where d stands for the smallest distance between the plane of the strip and the exciting sources. The method used in this approach is the classical Wiener–Hopf technique. Although our main purpose is to present the essentials of the theory, deferring its detailed application to some specific problems to later publications, it seems appropriate to include some representative examples to illustrate the new possibilities opened by this theory.

As is well known, the two-sided double Laplace transform technique, combined with the Wiener–Hopf procedure, reduces the abovementioned problem to a functional equation of the form (see 1, pp. 196–207 or Sect. 2, below)

$$e^{hs}\Phi_-(\sigma, s) + e^{-hs}\Phi_+(\sigma, s) + K(\sigma, s)P(\sigma, s) = -F(\sigma, s) \tag{3a}$$

or

$$\varphi_+(\sigma, s) = \frac{\lambda}{2\pi i}\int_{L^-} G(\sigma, \zeta)\frac{\varphi_+(\sigma, \zeta)}{\zeta + s}\, d\zeta + f(\sigma, s), \quad \lambda^2 = 1\,, \tag{3b}$$

where Φ_+, φ_+ and f are functions which are regular with respect to s in some right-half plane, B_+ say, of the complex s-plane. Likely, Φ_- is a regular function of s in some left-half plane B_-.

One supposes essentially that B_+ and B_- have a non-empty common intersection B. $P(\sigma, s)$ is an unknown integral function of s while F, f, G and K are given. Finally, the integration line L^- is an arbitrary straight line in B, parallel to the imaginary axis, such that the point $(-s)$ lies always in the left-hand side of L^-.

With regard to the solution of the functional equation (3b) some approximate methods are known [1–5]. Recently, the present author has given an exact method which can be applied in the general case to find the exact solution, if any [6]. In the general theory that we are going to establish for a dissipative medium, the equation (3b) being already approximate, the afore-mentioned general exact method is not applicable. So we give a new method for obtaining an approximate solution to (3b), especially suitable when the relation $1/\mathrm{Im}(k) \ll h$ is satisfied.

The essentials of the theory are given in Sect. 2, while Sect. 3 is devoted to some illustrative examples. Some relations concerning the far-field approximation and the edge condition are presented in Appendix 1 and Appendix 2.

2. Approximate theory in a dissipative medium

2.1. Hypotheses

In order to get a satisfactorily approximate theory we assume that the following requirements are fulfilled:

(i) The incident wave is monochromatic (the time factor $e^{-i\omega t}$ is suppressed throughout). Because of the radiation condition satisfied at infinity, the asymptotic expression of u for $r \to \infty$ contains a factor e^{ikr}.

(ii) The edge condition satisfied at the edges of S is of the form

$$u(0, y, z) \sim a(y)(z \pm h)^{\alpha} \text{ for } z \to \mp h,$$

where α is a real constant obeying the conditions

$$\lim_{s \to \infty} \frac{\mathscr{L}_s(i\gamma, \sigma, s)}{s^{2\alpha+2-\nu}} = 0, \quad \gamma^2 = s^2 + \sigma^2 + k^2, \tag{4}$$

$$\lim_{s \to \infty} \frac{\mathscr{L}_s(i\gamma, \sigma, s)}{s^{\nu-2}} = \infty, \tag{5}$$

with $\nu = +1$ or $\nu = 0$ in the case 2.2.1. or 2.2.2 below, respectively.

(iii) The operator $\mathscr{L}_s$ is even with respect to its third argument while it is even or odd with respect to the first:

$$\begin{aligned} &\mathscr{L}_s\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right) u \equiv \mathscr{L}_s\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) u \\ &\mathscr{L}_s\left(-\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) u \equiv (-1)^{1-\nu} \mathscr{L}_s\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) u. \end{aligned} \tag{6}$$

(iv) The imaginary part of k, Im (k), is sufficiently large: $\mathrm{Im}(hk) \gg 1$.

(v) The exciting sources are located in a domain sufficiently far from the plane of the strip, so that we can write

$$\psi(y, z) \simeq \psi(y, 0), \qquad z \in (-h, h)$$

if $\psi(y, 0) \not\equiv 0$. When $\psi(y, 0) \equiv 0$ this latter must be replaced by an averaged quantity, independent of z.

2.2. Reduction to a functional equation

Considering the hypothesis (i) and (iv) we conclude that the function u can be represented by

$$u(x,y,z)=\begin{cases}\displaystyle\int_{L_\sigma}\int_{L_s} A(\sigma,s)\,\mathrm{e}^{\mathrm{i}\gamma(\sigma,s)x+\sigma y+sz}\,ds\,d\sigma, & x>0\,,\\[2ex] \displaystyle\int_{L_\sigma}\int_{L_s} B(\sigma,s)\,\mathrm{e}^{-\mathrm{i}\gamma(\sigma,s)x+\sigma y+sz}\,ds\,d\sigma, & x<0\,,\end{cases} \tag{7}$$

where $A(\sigma, s)$ and $B(\sigma, s)$ are functions which will be determined with the aid of the other requirements. The integration line L_s is an arbitrary straight line—parallel to the imaginary axis—in the strip B defined by

$$\mathrm{Re}(\mathrm{i}k)<\mathrm{Re}(s)<\mathrm{Re}(-\mathrm{i}k)\,,$$

while L_σ is coincident with the imaginary axis, say L_0. The function $\gamma(\sigma, s)$ is defined on an appropriately cut s-plane by

$$\gamma(\sigma,s)=\{k^2+s^2+\sigma^2\}^{\frac{1}{2}},\quad \mathrm{Im}\,\gamma\geqq 0\,.$$

It has branch points at $\pm s(\sigma)$ with

$$s(\sigma)=\mathrm{i}\{k^2+\sigma^2\}^{\frac{1}{2}},\ \mathrm{Im}\,\{k^2+\sigma^2\}^{\frac{1}{2}}\geqq 0\,.$$

These points are all located outside the strip B and on the hyperbola

$$\mathrm{Im}(s)\cdot\mathrm{Re}(s)=-k_1k_2\,.$$

We assume that the s-plane is cut along parts of this hyperbola (see Fig. 1).

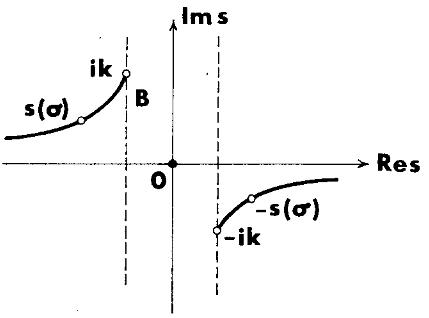

Figure 1.

By considering the boundary condition (2) and the continuity of u and grad u on $x=0, |z|>h$ one writes

$$\begin{aligned}&\textstyle\int\int [A-B]\,\mathrm{e}^{sz+\sigma y}ds\,d\sigma=0\,,\quad |z|>h\,,\\ &\textstyle\int\int [A+B]\gamma\,\mathrm{e}^{sz+\sigma y}ds\,d\sigma=0\,,\quad |z|>h\,,\\ &\textstyle\int\int A\mathscr{L}_s(\mathrm{i}\gamma,\sigma,s)\,\mathrm{e}^{sz+\sigma y}ds\,d\sigma=\psi(y,z)\,,\quad |z|<h\,,\\ &\textstyle\int\int [A\mathscr{L}_s(\mathrm{i}\gamma,\sigma,s)-B\mathscr{L}_s(-\mathrm{i}\gamma,\sigma,s)]\,\mathrm{e}^{sz+\sigma y}ds\,d\sigma=0\,,\quad |z|\lesseqgtr h\,.\end{aligned} \tag{8}$$

Note that the last equation of Eqns. (8) is valid for $|z|>h$, as well as for $|z|<h$, since the physical quantity corresponding to $\mathscr{L}_s u$ is continuous on $x=0$.

Now let us define

$$F(\sigma,s)=\frac{1}{(2\pi\mathrm{i})^2}\int_{-h}^{h}\mathrm{e}^{-sz}dz\int_{-\infty}^{\infty}\psi(y,z)\,\mathrm{e}^{-\sigma y}dy, \tag{9}$$

which enables us to write first

$$\textstyle\int\int F(\sigma,s)\,\mathrm{e}^{sz+\sigma y}ds\,d\sigma=\psi(y,z)\,,$$

and then to deduce from Eqns. (8)

$$
\begin{aligned}
&A-B=P_1(s)\,,\\
&[A+B]\gamma=P_2(s)\,,\\
&A\,\mathscr{L}_s(\mathrm{i}\gamma,\sigma,s)-F(\sigma,s)=\mathrm{e}^{hs}\Phi_-(s)+\mathrm{e}^{-hs}\Phi_+(s)\,,\\
&A\mathscr{L}_s(\mathrm{i}\gamma,\sigma,s)-B\mathscr{L}_s(-\mathrm{i}\gamma,\sigma,s)=0\,,
\end{aligned}
\tag{10}
$$

where P_1 and P_2 are two integral functions of s, Φ_+ is regular in the right-half plane B_+ defined by $\mathrm{Re}(s)>\mathrm{Re}(ik)$, except probably at infinity, while Φ_- is regular in B_- defined by $\mathrm{Re}(s)<\mathrm{Re}(-ik)$, except probably at infinity. Note that all of these functions depend also implicitly on the parameter σ and $\Phi_\pm(s)$ have algebraic behaviour for $s\to\infty$ (See Appendix 2).

Eliminate A and B from Eqns. (10) and consider the hypothesis (iii) to find

$$
\begin{aligned}
&[1+(-1)^{1-\nu}]P_1+[1-(-1)^{1-\nu}]\frac{P_2}{\gamma}=0\,,\\
&\frac{1}{2}\left\{P_1+\frac{P_2}{\gamma}\right\}\mathscr{L}_s(\mathrm{i}\gamma,\sigma,s)-F(\sigma,s)=\mathrm{e}^{hs}\Phi_-(s)+\mathrm{e}^{-hs}\Phi_+(s)\,.
\end{aligned}
\tag{11}
$$

Now it is natural to distinguish the following two cases:

2.2.1. The case $\nu=1$.

If the operator $\mathscr{L}_s$ is even with respect to its first argument then the first equation of Eqns. (11) yields $P_1\equiv 0$. Then the second equation of Eqns. (11) becomes

$$
\mathrm{e}^{hs}\Phi_-(s)+\mathrm{e}^{-hs}\Phi_+(s)-\frac{\mathscr{L}_s(\mathrm{i}\gamma,\sigma,s)}{2\gamma}P_2(s)=-F(\sigma,s)\,.
\tag{12}
$$

Hence the problem is reduced to find the functions $\Phi_\pm(s)$ which solve this latter equation. Once the functions $\Phi_\pm(s)$ are found, then A and B are determined by

$$
A(\sigma,s)=B(\sigma,s)=\{\mathrm{e}^{hs}\Phi_-+\mathrm{e}^{-hs}\Phi_++F\}/\mathscr{L}_s(\mathrm{i}\gamma,\sigma,s)\,.
$$

2.2.2. The Case $\nu=0$.

In this case, proceeding as before, one gets

$$
\begin{aligned}
&P_2\equiv 0\,,\\
&\mathrm{e}^{sh}\Phi_-(s)+\mathrm{e}^{-hs}\Phi_+(s)-\tfrac{1}{2}\mathscr{L}_s(\mathrm{i}\gamma,\sigma,s)P_1=-F(\sigma,s)\,,\\
&A(\sigma,s)=-B(\sigma,s)=\{\mathrm{e}^{hs}\Phi_-+\mathrm{e}^{-hs}\Phi_++F\}/\mathscr{L}_s(\mathrm{i}\gamma,\sigma,s)\,.
\end{aligned}
\tag{13}
$$

Note that both the Eqns. (12) and (13) are of the type (3a) with

$$
K(s)=-\tfrac{1}{2}\mathscr{L}_s(\mathrm{i}\gamma,\sigma,s)[\gamma(\sigma,s)]^{-\nu}\,,
\tag{14}
$$

and, because of Eqns. (6), one always has

$$
K(s)\equiv K(-s)\,.
\tag{15}
$$

Since the symmetry condition (15) is met, Eqn. (12) or Eqn. (13) can be reduced to an equation of the form (see, for example, [1], pp. 196–199)

$$
\varphi_+(s)-\frac{\lambda}{2\pi\mathrm{i}}\int_{L^-}G(\zeta)\frac{\varphi_+(\zeta)}{\zeta+s}\,d\zeta=f(s)\,,\quad \lambda=\pm1\,,
\tag{16}
$$

where L^- is a straight-line in B, parallel to the imaginary axis so that the point $\zeta=-s$ lies always in the left-hand side of L^-. $\varphi_+(s)$ and $f(s)$ are given by

$$\varphi_+(s) = \frac{\Phi_+(s) - \lambda\Phi_-(-s)}{K_+(s)},$$

$$f(s) = \frac{\lambda}{2\pi i} \int_{L^-} \frac{[F(\zeta) - \lambda F(-\zeta)]\, e^{-\zeta h}}{K_-(\zeta)(\zeta + s)} d\zeta, \tag{17}$$

while $K_\pm(s)$ and G are defined as follows:

$$K(s) \equiv K_+(s)K_-(s), \ K_-(-s) \equiv K_+(s), \ G(\zeta) = \frac{K_+(\zeta)}{K_-(\zeta)} e^{-2\zeta h}.$$

By definition, $K_+(s)$ (or $K_-(s)$) has neither singularities nor zeros in B_+ (or B_-).

It is worthwhile to point out that in order to reduce Eqn. (12) or (13) to Eqn. (16) one should have

$$\text{order } \Phi_+(s) < \text{order } K_+(s) \text{ for } s \to \infty \text{ in } \mathrm{Re}(s) > \mathrm{Re}(ik).$$

As it will be shown in Appendix 2, this yields the condition (4) of Section 2, 2.1.

2.3. *Approximate solution of functional equation.*

Consider now the hypothesis (v). Because of it $F(\sigma, s)$ in Eqn. (9) and, consequently, $f(s)$ in Eqns. (17) can be replaced by

$$F(\sigma, s) = \frac{2D(\sigma)}{(2\pi i)^2} \frac{\sinh sh}{s}, \ D(\sigma) = \int_{-\infty}^{\infty} \psi(y, 0)\, e^{-\sigma y} dy$$

and

$$f(s) = \frac{\lambda(1-\lambda)}{(2\pi i)^3} D(\sigma) \int_{L^-} \frac{1 - e^{-2\zeta h}}{\zeta(\zeta + s)K_-(\zeta)} d\zeta. \tag{18}$$

With $\lambda = +1$, Eqns. (18) and (16) give $f(s) \equiv 0$, so $\varphi_+(s) \equiv 0$ and consequently

$$\Phi_+(s) \equiv \Phi_-(-s).$$

With $\lambda = -1$ (Eqn. (16)) becomes an equation by which one determines the function $\Phi_+(s)$ (in what follows we replace λ by -1). Since the substitution of Eqn. (18) into Eqn. (16) makes it approximate, the aforementioned exact solution method [6] is not applicable. We therefore attempt to develope a new method to obtain an approximate solution.

Equation (16) is a functional equation of the form

$$\mathcal{L}\varphi_+ = f(s), \qquad s \in B_+.$$

Now, take into account Eqn. (5) which shows that one has [see Appendix 2]

$$\frac{1}{sK_-(s)} = o(1) \text{ for } s \to \infty, \qquad s \in B_-, \tag{19}$$

and replace φ_+ by

$$\varphi(s) = -\frac{D(\sigma)}{2\pi^2} \left[\frac{1}{K_+(s)} - \frac{1}{K_+(0)} \right] \frac{1}{s}$$

to get

$$\mathcal{L}\varphi = f(s) + f_1(s),$$

with

$$f_1(s) = \frac{D(\sigma)}{2\pi^2 K_+(0)} \frac{1}{2\pi i} \int_{L^-} \frac{K_+(\zeta)}{K_-(\zeta)} \frac{e^{-2\zeta h}}{\zeta(\zeta + s)} d\zeta.$$

Where the point $\zeta = 0$ as well as $\zeta = -s$ lies in the left-hand side of L^-. This shows that when $f_1(s)$

can be neglected with respect to $f(s)$, the function $\varphi(s)$ is an approximate solution to Eqn. (16) with $\lambda = -1$.

With the aid of Jordan's lemma we can convert $f_1(s)$ into a sum of residues and branch-cut integrals in B_+. Each of these terms contains a factor $e^{-2\chi h}$, with $\mathrm{Re}(\chi) > \mathrm{Im}(k)$, which, on the basis of (iv), shows that this sum is negligibly small. Then Eqns. (17) yield approximately

$$\Phi_+(s) \equiv \Phi_-(-s) = -\frac{D(\sigma)}{4\pi^2}\left[1 - \frac{K_+(s)}{K_+(0)}\right]\frac{1}{s}. \tag{20}$$

For the far-field approximation we need only the values of Φ_+ at

$$\sigma = \sigma_0 \equiv ik \sin\theta \sin\varphi\,, \quad s = s_0 \equiv ik\cos\theta\,,$$

where R, θ and φ stand for the usual spherical co-ordinates (see Appendix 1).

3. Some illustrative examples

Suppose that S is a unidirectionally conducting strip and the incident field is produced by a Hertzian dipole located at $(d, 0, 0)$. We consider only three different cases illustrated in Fig. 2.

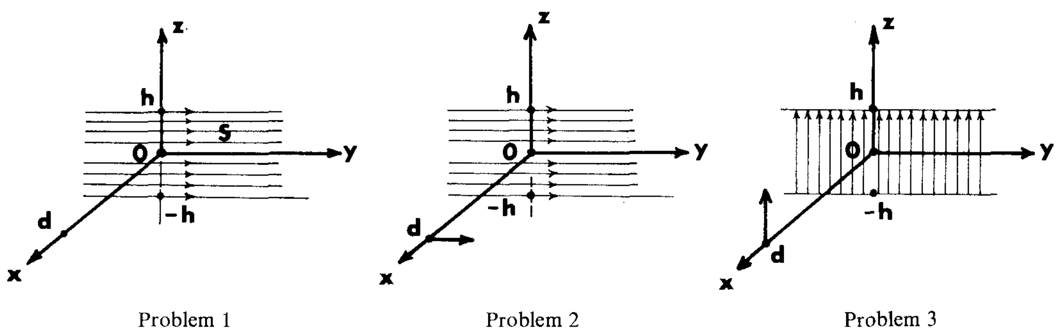

Figure 2

In Problem 1 the strip is longitudinally conducting while the dipole moment is $Il\mathbf{e}_x$. In the Problem 2 the strip remains unchanged but the dipole moment is now $Il\mathbf{e}_y$. In the Problem 3 the strip is transversally conducting while the dipole has a moment $Il\mathbf{e}_z$. In all of these problems the function u stands for the non-zero component of the vector potential due to currents induced on the strip. In the case of Problem 1 and Problem 2 one has

$$\mathscr{L}_s u \equiv \left(k^2 + \frac{\partial^2}{\partial y^2}\right)u, \qquad \nu=1,\ \alpha=0\,,$$

$$\psi(y,z) \equiv -\frac{k^2}{i\omega}E_y^{\mathrm{inc}}(0,y,z)\,,$$

while in the case of Problem 3

$$\mathscr{L}_s u \equiv \left(k^2 + \frac{\partial^2}{\partial z^2}\right)u, \qquad \nu=1,\ \alpha=\tfrac{3}{2}\,,$$

$$\psi(y,z) \equiv -\frac{k^2}{i\omega}E_z^{\mathrm{inc}}(0,y,z)\,.$$

This shows that the conditions (4) and (5) are fulfilled. When the relations $1/\mathrm{Im}(k) \ll h \ll d$ are also satisfied, the above theory can be applied. The following tables summarize the results of step-by-step application of the theory. Notice that since in Problems 1 and 2 the factor $-\frac{1}{2}$-$\mathscr{L}_s(i\gamma, \sigma, s)$ is constant with respect to s, it can be introduced into $P_2(s)$. The same is true for the factor $-1/2$ in the case of Problem 3.

In what follows μ denotes the magnetic permeability of the medium and superscript "inc" stands for "incident".

pr.	$\psi(y, z)$
1	$-\dfrac{k^2}{4\pi}\mu Il\,dy \exp\{ik(d^2+y^2+z^2)^{\frac{1}{2}}\}\cdot\{d^2+y^2+z^2\}^{-\frac{3}{2}}$
2	$-\dfrac{k^2}{4\pi}\mu Il \exp\{ik(d^2+y^2+z^2)^{\frac{1}{2}}\}\cdot\{d^2+y^2+z^2\}^{-\frac{1}{2}}\left(1-\dfrac{y^2}{d^2+y^2}\right)$
3	$-\dfrac{k^2}{4\pi}\mu Il \exp\{ik(d^2+y^2+z^2)^{\frac{1}{2}}\}\cdot\{d^2+y^2+z^2\}^{-\frac{1}{2}}\left(1-\dfrac{z^2}{d^2+y^2+z^2}\right)$

pr.	$D(\sigma)$
1	$-\dfrac{\mu Il\,dk^2}{4\pi}\displaystyle\int_{-\infty}^{\infty}\frac{e^{ik(d^2+y^2)^{1/2}}}{\{d^2+y^2\}^{\frac{3}{2}}}e^{-\sigma y}y\,dy$
2	$-\dfrac{\mu Ilk^2}{4\pi}d^2\displaystyle\int_{-\infty}^{\infty}\frac{e^{ik(d^2+y^2)^{1/2}}}{\{d^2+y^2\}^{2}}e^{-\sigma y}dy$
3	$-\dfrac{\mu Ilk^2}{4\pi}\displaystyle\int_{-\infty}^{\infty}\frac{e^{ik(d^2+y^2)^{1/2}}}{(d^2+y^2)^{\frac{1}{2}}}e^{-\sigma y}dy \equiv -\frac{\mu Ilk^2}{2\pi}K_0(d(-\sigma^2-k^2)^{\frac{1}{2}}\}$

K_0: Mc Donald's function [7].

pr.	$\mathscr{L}_s(i\gamma, \sigma, s)$	$K_+(s)$	$\varphi(s)$
1	$k^2+\sigma^2\to -2$	$e^{-i\pi/4}/\{s-s(\sigma)\}^{\frac{1}{2}}$	$-\dfrac{D(\sigma)}{2\pi^2}e^{i\pi/4}\{s-s(\sigma)\}^{\frac{1}{2}}$
2	$k^2+\sigma^2\to -2$	$e^{-i\pi/4}/\{s-s(\sigma)\}^{\frac{1}{2}}$	$-\dfrac{D(\sigma)}{2\pi^2}e^{i\pi/4}\{s-s(\sigma)\}^{\frac{1}{2}}$
3	$k^2+s^2\to -2(k^2+s^2)$	$\dfrac{e^{-i\pi/4}(k+is)}{\{s-s(\sigma)\}^{\frac{1}{2}}}$	$-\dfrac{D(\sigma)}{2\pi^2}e^{i\pi/4}\dfrac{\{s-s(\sigma)\}^{\frac{1}{2}}}{k+is}$

pr.	$A(\sigma, s)$
1	$\dfrac{-D(\sigma)\{-s(\sigma)\}^{\frac{1}{2}}}{4\pi^2(k^2+\sigma^2)}\cdot\dfrac{1}{s}\left\{\dfrac{e^{hs}}{[-s-s(\sigma)]^{\frac{1}{2}}}-\dfrac{e^{-hs}}{[s-s(\sigma)]^{\frac{1}{2}}}\right\}$
2	$\dfrac{-D(\sigma)\{-s(\sigma)\}^{\frac{1}{2}}}{4\pi^2(k^2+\sigma^2)}\cdot\dfrac{1}{s}\left\{\dfrac{e^{hs}}{[-s-s(\sigma)]^{\frac{1}{2}}}-\dfrac{e^{-hs}}{[s-s(\sigma)]^{\frac{1}{2}}}\right\}$
3	$\dfrac{-D(\sigma)\{-s(\sigma)\}^{\frac{1}{2}}}{4\pi^2k(k^2+s^2)}\cdot\dfrac{1}{s}\left\{\dfrac{(k-is)e^{+hs}}{[-s-(\sigma)]^{\frac{1}{2}}}-\dfrac{(k+is)e^{-hs}}{[s-s(\sigma)]^{\frac{1}{2}}}\right\}$

pr.	far-field approximation of the scattered field ($\sigma_0 = ik \sin\theta \sin\varphi$, $s_0 = ik \cos\theta$, $\gamma_0 = \gamma(\sigma_0, s_0) = k \sin\theta \cos\varphi$)	
1	$E_x^s = \dfrac{-\omega\gamma_0\sigma_0}{k^2} A(\sigma_0, s_0)$	$B_x^s = -s_0 A(\sigma_0, s_0)$
	$E_y^s = i\omega\left(1 + \dfrac{\sigma_0^2}{k^2}\right) A(\sigma_0, s_0)$	$B_y^s = 0$
	$E_z^s = \dfrac{i\omega}{k^2} \sigma_0 s_0 A(\sigma_0, s_0)$	$B_z^s = i\gamma_0 A(\sigma_0, s_0)$
2	$E_x^s = \dfrac{-\omega\gamma_0\sigma_0}{k^2} A(\sigma_0, s_0)$	$B_x^s = -s_0 A(\sigma_0, s_0)$
	$E_y^s = i\omega\left(1 + \dfrac{\sigma_0^2}{k^2}\right) A(\sigma_0, s_0)$	$B_y^s = 0$
	$E_z^s = \dfrac{i\omega}{k^2} \sigma_0 s_0 A(\sigma_0, s_0)$	$B_z^s = i\gamma_0 A(\sigma_0, s_0)$
3	$E_x^s = \dfrac{-\omega}{k^2} \gamma_0 s_0 A(\sigma_0, s_0)$	$B_x^s = \sigma_0 A(\sigma_0, s_0)$
	$E_y^s = \dfrac{i\omega}{k^2} \sigma_0 s_0 A(\sigma_0, s_0)$	$B_y^s = -i\gamma_0 A(\sigma_0, s_0)$
	$E_z^s = i\omega\left(1 + \dfrac{s_0^2}{k^2}\right) A(\sigma_0, s_0)$	$B_z^s = 0$

4. Conclusions

The present study shows that a satisfactorily approximate theory of the scalar scattering by a strip in a dissipative medium can be established when the relations $1/\mathrm{Im}(k) \ll h \ll d$ are satisfied.

The expressions valid in the far-field approximation are obtained by simple manipulations provided that the integral associated with $D(\sigma)$ can be evaluated.

It is evident that after some simple modification the above theory can also be applied in the case of plane incident wave when the hypothesis (iii) is fulfilled. The Babinet's complementarity principle permits to apply the above results to the infinite slit problems.

It is also evident that the method used here can be applied when cylindrical structures other than the strip are concerned.

Appendix 1. Far-field approximation.

Let us consider Eqn. (7) for $x > 0$ and substitute x, y, z by usual spherical co-ordinate R, θ, φ. Then we write

$$u = \int_{L_0} e^{R\sigma \sin\theta \sin\varphi} \left[\int_{L_s} A(\sigma, s)\, e^{R\{i\gamma(\sigma,s)\sin\theta\cos\varphi + s\cos\theta\}} ds \right] d\sigma\,. \tag{A.1}$$

By applying the classical saddle-point method [8; pp. 293–312] we get a far-field approximation to the inner integral

$$B(\sigma) = \int_{L_s} A(\sigma, s)\, e^{R\{i\gamma(\sigma,s)\sin\theta\cos\varphi + s\cos\theta\}} ds$$

as follows:

$$B(\sigma) \sim \{2\pi i(k^2+\sigma^2)^{\frac{1}{2}}\}^{\frac{1}{2}} \frac{e^{-i(k^2+\sigma^2)^{1/2} R\cos\theta/\cos w_0}}{(-R\cos\theta/\cos w_0)^{\frac{1}{2}}} A\{\sigma, -i(k^2+\sigma^2)^{\frac{1}{2}}\cos^2 w_0\} \sin w_0\,,$$

where $(k^2+\sigma^2)^{\frac{1}{2}}$ denotes the branch which is reduced to k for $\sigma \to 0$ in the complex σ-plane cut along lines which connect the points $\pm ik$ to infinity without penetrating into the domain $\mathrm{Re}(+ik) < \mathrm{Re}(\sigma) < R(-ik)$, while w_0 is defined by

$$\mathrm{tg}\, w_0 = -\mathrm{tg}\,\theta \cos\varphi, \qquad w_0 \in [\pi-\theta, \pi]\,. \tag{A.2}$$

We insert the above expression of $B(\sigma)$ into Eqn. (A.1) to obtain

$$u \sim \left(\frac{2\pi}{R\cos\theta}\right)^{\frac{1}{2}} \sin w_0 \int_{L_0} s_0^{\frac{1}{2}} A(\sigma, s_0) \exp\left[R\left\{-i(k^2+\sigma^2)^{\frac{1}{2}}\frac{\cos\theta}{\cos w_0} + \sigma\sin\theta\sin\varphi\right\}\right] d\sigma, \tag{A.3}$$

with

$$s_0 = -i(k^2+\sigma^2)^{\frac{1}{2}}\cos w_0\,. \tag{A.4}$$

We apply now once again the saddle-point method to Eqn. (A.3). The saddle-point σ_0 is given by

$$\sigma_0 \cos\theta - s_0 \sin\theta \sin\varphi = 0\,. \tag{A.5}$$

From Eqns. (A.2), (A.4) and (A.5) one gets

$$\sigma_0 = ik\sin\theta\sin\varphi\,, \quad s_0 = ik\cos\theta$$

and then

$$u \sim 2\pi ik A(ik\sin\theta\sin\varphi,\ ik\cos\theta)\sin\theta\cos\varphi\ \frac{e^{ikR}}{R}, \quad (\cos\varphi > 0)\,. \tag{A.6}$$

Note that by reducing the integration line L_0 into the steepest-descent line through the saddle-point σ_0 one assumed that no singular point of the integrand is crossed. This fact must be checked in each example. Otherwise the necessary changes must be made in (A.6).

The expressions for $x < 0$ can be obtained from the symmetry relations:

$$u(-x, y, z) \equiv (-1)^{1-\nu} u(x, y, z)\,.$$

Appendix 2. On the conditions (4) and (5).

According to the special choice of $F(\sigma, s)$, Φ_+ is nothing but the double-Laplace transform of $[\mathscr{L}_s u(x, y, z+h) I(z)]_{x=0}$, where $I(z)$ stands for the Heaviside unit function:

$$e^{-hs}\Phi_+(s) = \int_{-\infty}^{\infty} \left\{\int_h^{\infty} [\mathscr{L}_s u(x, y, z) e^{-sz}]_{x=0}\, dz\right\} e^{-\sigma y}\, dy\,.$$

Hence, if one has

$$[\mathscr{L}_s u]_{x=0} \sim c(y)(z-h)^{\beta} \text{ for } z \to h,\ z-h > 0$$

then one writes

$$\Phi_+(s) \sim C(\sigma)\frac{1}{s^{\beta+1}} \text{ for } s \to \infty\,, \quad \mathrm{Re}(s) > \mathrm{Re}(ik)\,. \tag{A.7}$$

On the other hand, (ii) gives

$$\text{double Tr. of } u(0, y, z+h) I(z) \sim A(\sigma)\frac{1}{s^{\alpha+1}} \text{ for } s \to \infty,\ \mathrm{Re}(s) < \mathrm{Re}(ik)$$

and then

$$\text{double Tr. of } [\mathscr{L}_s u(x, y, z+h) I(z)]_{x=0} \sim \mathscr{L}_s(\mathrm{i}\gamma, \sigma, s) A(\sigma) \frac{1}{s^{\alpha+1}}$$

This shows that

$$\frac{C(\sigma)}{s^{\beta+1}} \sim A(\sigma) \cdot \frac{\mathscr{L}_s(\mathrm{i}\gamma, \sigma, s)}{s^{\alpha+1}} \tag{A.8}$$

(The same line of arguments can be repeated for Φ_-). In order to reduce Eqn. (12) or (13) to an equation of the type (16) we must have

$$\text{order } \Phi_+ < \text{order } K_+ = \tfrac{1}{2} \text{ order } \frac{\mathscr{L}_s(\mathrm{i}\gamma, \sigma, s)}{\gamma^\nu}$$

$$= \tfrac{1}{2} \text{ order } \mathscr{L}_s(\mathrm{i}\gamma, \sigma, s) - \frac{\nu}{2}.$$

By considering also (A.7) and (A.8) this yields the condition (4) of Sect. 2, 2.2.1.

Now consider the relation (19). It shows that one has

$$K_\pm(s) = O(s^{\varepsilon-1})$$

with some $\varepsilon > 0$. Hence Eqn. (14) gives

$$\frac{\mathscr{L}_s(\mathrm{i}\gamma, \sigma, s)}{\gamma^\nu} = O(s^{2\varepsilon-2})$$

which is nothing but the relation (5).

REFERENCES

[1] B. Noble, *Methods based on the Wiener–Hopf technique*, Pergamon Press, N.Y. (1968).
[2] R. F. Millar, Diffraction by a wide slit and complementary strip, *Proc. Camb. Phil. Soc.*, 54 (1958) 479–511.
[3] S. R. Seshadri and T. T. Wu, High-frequency diffraction of plane waves by an infinite slit for grazing incidence, *Trans. IRE* AP-8(1) (1960) 37–42.
[4] D. S. Jones, *The theory of electromagnetism*, Pergamon Press, N.Y. (1964).
[5] R. P. Kanwal and B. K. Sachdeva, Approximate solutions of certain integral equations for the diffraction by two strips, *ZAMP* 24 (1973) 111–119.
[6] M. Idemen, On the functional equation related to the three-part mixed boundary value problems, to appear in *SIAM J. Appl. Math.*, 27 (3) (1974).
[7] A. Erdélyi, W. Magnus, F. Oberhettingen and F. G. Tricomi, *Tables of Integral Transforms*, Vol. I, p. 17, no 27, McGraw-Hill (1954), N.Y.
[8] W. I. Smirnow, *A course of Higher Mathematics*, III, 2, Pergamon Press, N.Y. (1964) 293–312.